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REPORT

Correction of primary chromaticities of display tubes by matrixing of gamma-corrected RGB signals

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Summary

Signal origination equipment produces signals which give optimum colour fidelity using a given set of display primaries. These are the C.C.I.R. recommended primaries adopted for P.A.L. System I colour television.

The use of display primaries with incorrect chromaticities gives rise to colour errors that can be unacceptable. Linear colour-separation signals are not normally available in a picture monitor so simple matrix correction cannot theoretically be used. Matrixing of gamma-corrected signals can however provide a compromise solution.

The colour errors incurred by matrixing are discussed and two methods of deriving a suitable matrix are described.

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1. Introduction

In order that a colour television system should have good colour fidelity, the colour analysis must be matched to the display-primary chromaticities. In 1953, the N.T.S.C. recommended a set of primary chromaticites representative of the best phosphors available at the time. Subsequently, developments in phosphor materials were directed towards increased brightness; as a consequence of this, the primary chromaticities of most tubes were considerably shifted from the original N.T.S.C. values.

Colour rendering was not seriously impaired by this shift because the camera analysis characteristics in use at the time were based upon only the positive lobes of the ideal analysis curves, which are not seriously affected by the choice of display primary chromaticities.

With the advent of linear matrixing for camera analysis in the late 1960's, display-primary chromaticities had to be more closely specified because the negative lobes of the camera-analysis characteristics thus introduced are closely dependent upon them.

A working party was established by the E.B.U. to investigate the situation,[1,2] resulting in a set of display primaries being adopted for P.A.L. System I colour television.[3] Cameras were subsequently designed to provide colour signals suited to these primaries.

Standard tolerances have been set on the System I display chromaticities for monitors which, on a chromaticity diagram consist of three "boxes" surrounding the ideal primaries.[4] The boxes were chosen on the basis of intrinsic primary colour, and the tolerances that could reasonably be expected in practice. A combination of primary chromaticities, lying within these boxes, is acceptable if it will reproduce a prescribed skin colour with less than a given error. This technique offers a convenient method of evaluating a display and is described elsewhere.

Further developments in tube phosphors unfortunately produced additional changes with a pronounced trend in a shift of the red and green primaries towards each other and there is growing concern that near-System I tubes may not be readily available to the broadcaster in the future.

A similar, but more serious, problem has already developed in the U.S.A. where the colour analysis of some cameras has been based on N.T.S.C. primaries but many domestic receivers and professional monitors are now equipped with modern tubes with very different primaries. Correction matrices at the display have been proposed[6,7,8] and to some extent have been put into practice but have not been completely successful.

In order to provide a solution, primarily for the broadcaster, to the potential problem in the U.K. and Europe, further investigations into matrix correction have been carried out. Ideally, a suitable method would allow television monitors to provide correct colour reproduction independently of the primaries available in future tubes. It could also be adapted to cope with the same problem in domestic receiver displays.

In the case of professional colour monitors a matrix operating on direct or decoded R'.G'.B'. colour-separation signals is probably most convenient. In the case of domestic receivers it may well be more economical to apply the matrix to the decoded colour-difference signals; methods of calculating the appropriate matrix are described in the Appendix.

2. Colorimetry background

2.1. Tristimulus colorimetry

In the tristimulus system of colorimetry any colour can be specified by matching it with a colour formed by a combination of three known primary colours, usually red, green and blue. According to Grassman's laws[9] the specification of a colour made up of the sum of any number of component colours is obtained by summing the specifications of the components. As a result of this the equivalent colour stimuli for an observer can be calculated from the tristimulus specifications of the spectrum colours because any colour may be considered to be the sum of a number of spectral colours. Sets of curves which give tristimulus specifications of the spectrum for an observer are called colour-mixture curves.

These curves give the amounts of three primaries needed for an observer to match all the spectrum colours. Different primaries produce a different set of curves and all colour mixture curves for an observer can be computed from any one set of colour-mixture curves for that observer.

The colour-mixture curves corresponding to a set of real primaries require negative values at some wavelengths in order to provide a match with some spectral colours. For greater convenience of computation the C.I.E. standarised, in 1931, on the set of $\bar{x}, \bar{y}$ and $\bar{z}$ colour mixture curves shown in Fig. 1. These curves are based on three fictitious primaries X, Y and Z chosen so as to eliminate negative values. They are derived by a transformation from experimental colour-mixture curves obtained with a set of physically realisable primaries and many observers (representing the standard observer).

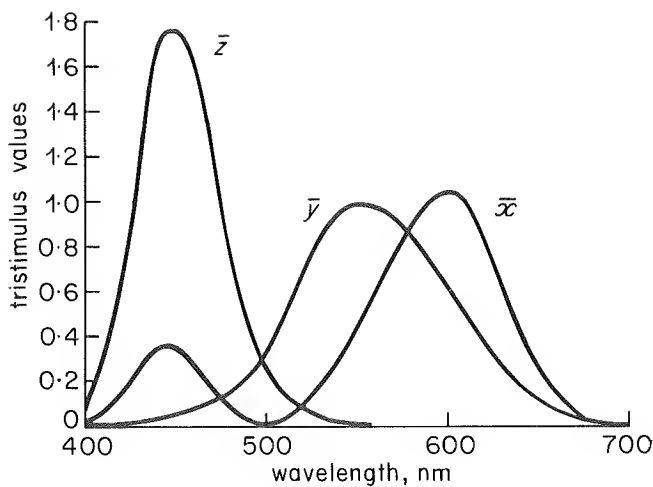The tristimulus values $(X, Y$ and $Z)$ for any colour

Fig. 1 - Standard C.I.E. colour-mixture curves

are found by integrating $E(\lambda)$, the spectral energy distribution of the colour wavelength-by-wavelength with the corresponding values from the standard observer curves shown in Fig. 1, as follows

$$X = \int_{\lambda_1}^{\lambda_2} E(\lambda).\bar{x}(\lambda).d\lambda$$

$$Y = \int_{\lambda_1}^{\lambda_2} E(\lambda).\bar{y}(\lambda).d\lambda$$

$$Z = \int_{\lambda_1}^{\lambda_2} E(\lambda).\bar{z}(\lambda).d\lambda$$

where $\lambda_1 = 380$ nm and $\lambda_2 = 770$ nm

The relative values of the functions $\bar{x}$, $\bar{y}$ and $\bar{z}$ are such that if $E(\lambda)$ has equal energy distribution then

$$X = Y = Z;$$

this corresponds to equal energy white.

2.2. Chromaticity diagrams

If, in Fig. 2 the vectors representing the C.I.E. primaries $(X, Y$ and $Z)$ are plotted along mutually perpendicular axes in space then the chromaticities of all realisable colours fall within the X, Y, Z primary triangle

in the unit plane where

$$X + Y + Z = 1$$

The projection of this plane onto the XY plane forms the 1931 (x,y) C.I.E. chromaticity diagram. A colour space vector corresponding to tristimulus values X, Y and Z intersects the unit plane at P and this point projected normally meets the XY plane at Q. The colour can thus be represented by the chromaticity co-ordinates (x,y) for Q on the C.I.E. chromaticity diagrams where

$$x = \frac{X}{X+Y+Z} \ , \quad y = \frac{Y}{X+Y+Z} \quad \text{and } z = \frac{Z}{X+Y+Z}$$

$$\text{and} \quad x + y + z = 1$$

The dominant wavelength and purity of a colour are thus uniquely specified by x and y. Its luminance is described by the tristimulus value Y.

On this diagram equal distances do not represent equally perceptible chromaticity differences, as shown in Fig. 3. A transformation can be made from this diagram to one on which perceptible colour differences are more uniformly represented. In this system the axes are the tristimulus values $(U, V$ and $W)$ which are related to X, Y and Z by the following matrix

$$\begin{bmatrix} U \\ V \\ W \end{bmatrix} = \begin{bmatrix} 0{\cdot}67 & 0 & 0 \\ 0 & 1{\cdot}00 & 0 \\ -0{\cdot}50 & 1{\cdot}50 & 0{\cdot}50 \end{bmatrix} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix}$$

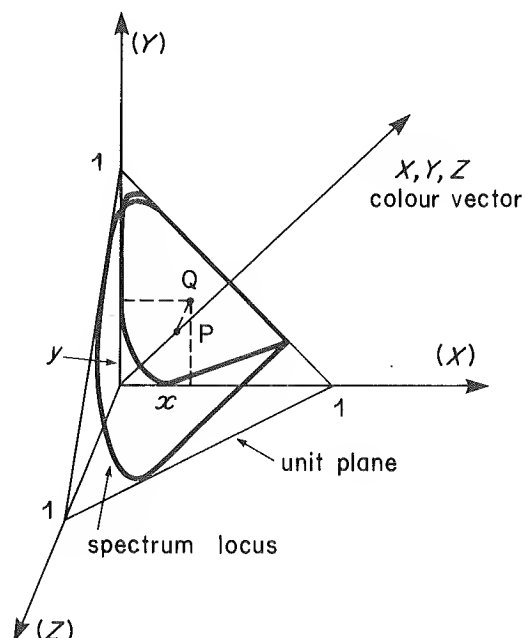The co-ordinates of the colour vector are normalised

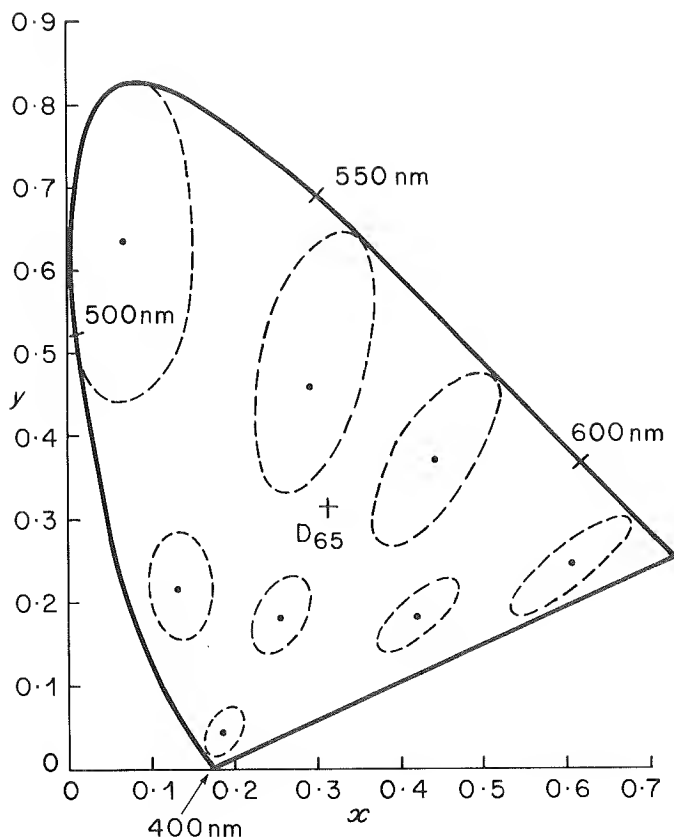*Fig. 2 - Tristimulus values and chromaticity co-ordinates in XYZ system*

Fig. 3 - C.I.E. 1931 x,y chromaticity diagram showing approximate perceptibility of chromaticity differences. Each ellipse shows 100 times the minimum perceptible change.

to produce the 1960 U.C.S. (uniform chromaticity scale) diagram shown in Fig. 4, which represents a considerable improvement in uniformity of chromaticity-difference perceptibility when compared with Fig. 3. The relationship between the tristimulus values and the chromaticity co-ordinates are as follows,

$$u = \frac{U}{U+V+W} \, , \quad v = \frac{V}{U+V+W} \quad \text{and} \quad w = \frac{W}{U+V+W}$$

Thus the chromaticity of a colour is defined by (u,v) and its luminance is described by the tristimulus value V (usually given the symbol L).

In 1964 the C.I.E. introduced a modified form[10] of this diagram which takes into account other perceptual effects such as saturation changes with brightness and applications to $10°$ visual fields rather than $2°$ fields as assumed here. More recently a Cieluv 76 formula[11] has been introduced which implies a new "nearly uniform" diagram and this may be used in future work.

2.3. Display matrices

The tristimulus values of the three primary colours produced by the display phosphors are

(U_R, V_R, W_R) for red,
(U_G, V_G, W_G) for green,
and (U_B, V_B, W_B) for blue

so that the tristimulus values (Uc, Vc, Wc) of the displayed colour corresponding to the red, green and blue input voltages (Rc, Gc, Bc) can be obtained from the matrix equation

$$\begin{bmatrix} Uc \\ Vc \\ Wc \end{bmatrix} = \begin{bmatrix} K_1 U_R & K_2 U_G & K_3 U_B \\ K_1 V_R & K_2 V_G & K_3 V_B \\ K_1 W_R & K_2 W_G & K_3 W_B \end{bmatrix} \begin{bmatrix} Rc \\ Gc \\ Bc \end{bmatrix}$$

The constants K_1, K_2 and K_3 represent weighting values, which take into account the display channel gains, and the luminous efficiency of the three primary phosphors which will generally be different; thus a form of normalisation must be introduced. The constants are derived by defining a display white point (illuminant D65) for which the corresponding red, green and blue drive voltages are equal. It represents the spectral distribution of a particular phase of daylight and has a correlated colour temperature of approximately $6504°K$. In the 1960 C.I.E. system the chromaticity co-ordinates of D_{65} are

$$u = 0.198, \qquad v = 0.312, \qquad \text{and} \qquad w = 0.490$$

2.4. Camera design

In order to reproduce correctly all those colours in the original scene which lie within the colour triangle of the display-tube primaries, the tristimulus values of the original and reproduced scene must be identical. The theoretically correct spectral sensitivity of a camera is thus derived from the tristimulus values of the display primaries. It can be shown[8] that the required red, green and blue channel voltages (R_c, G_c, B_c) from the camera can be calculated from the inverse of the above display-primaries tristimulus-matrix, as follows:

$$\begin{bmatrix} R_c \\ G_c \\ B_c \end{bmatrix} = \begin{bmatrix} K_1 U_R & K_2 U_G & K_3 U_B \\ K_1 V_R & K_2 V_G & K_3 V_B \\ K_1 W_R & K_2 W_G & K_3 W_B \end{bmatrix}^{-1} \begin{bmatrix} U \\ V \\ W \end{bmatrix}$$
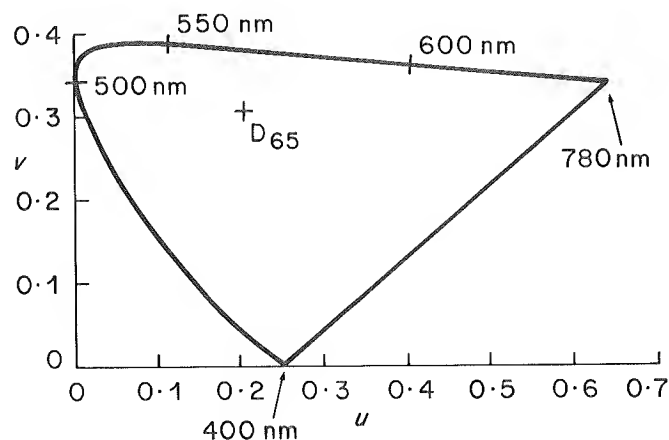

Fig. 4 - C.I.E. 1960 nearly uniform chromaticity diagram

and substituting, for U, V and W, the tristimulus values $u(\lambda)$, $v(\lambda)$ and $w(\lambda)$ respectively, (which correspond to the transformed UVW values of the standard XYZ colour matching curves).

If the display primaries were the same as the (UVW) primaries the required camera analysis would be the same as the $(U\ V\ W)$ transforms of the (XYZ) colour-mixture curves shown in Fig. 1. Camera analysis curves based on E.B.U. display primaries are shown in Fig. 5. It will be seen that these curves have negative lobes which correspond approximately in magnitude and spectral location to the areas of the chromaticity diagram lying within the spectrum locus, but outside the colour triangle of the display primaries. The negative responses are required for the correct reproduction of any colour, however camera tubes cannot produce negative signals corresponding to these lobes but linear matrix correction of the red, green and blue linear outputs signals from the tubes can be employed to approximate the required characteristics.

2.5. Gamma correction

The light output from a television display is related to the drive voltage by a power law with an exponent termed the gamma of the tube; its value is usually about 2·7. The output of most modern camera tubes is very nearly proportional to light input so that correction to compensate for the display must be applied in the camera and takes the form of a non-linear amplifier known as a gamma corrector. Theoretically it would be

expected that the gamma corrector exponent would be the reciprocal of the display gamma but it is normally made slightly higher than this, due to the limitations imposed by camera noise and tube lag. It is also thought that due to physiological effects, an overall system gamma slightly greater than unity is preferred subjectively.

2.6. Error evaluation

The unit used in this Report to quantify chromaticity differences, is the "just noticeable difference"[12] or j.n.d. and has a magnitude of approximately 0·004 units on the 1960 C.I.E. chromaticity diagram. This is considered to be an appropriate unit under typical television viewing conditions. However, skin-tone is an important exception as the brain appears to have a very accurate memory reference as to its chromaticity and luminance. Thus for skin-tones and other important colours a smaller sized unit (in the region of 0·002 uv units) is more appropriate.

The luminance j.n.d. is a 2% difference in luminance between adjacent identical colours, although this unit is considered to be too large for low luminance levels.

3. Linear matrix correction

The effect of a phosphor chromaticity shift in a linear system (display-tube gamma = 1) can be explained by considering the relative amplitudes of the colour-separation signals when only one of the phosphor chroma-
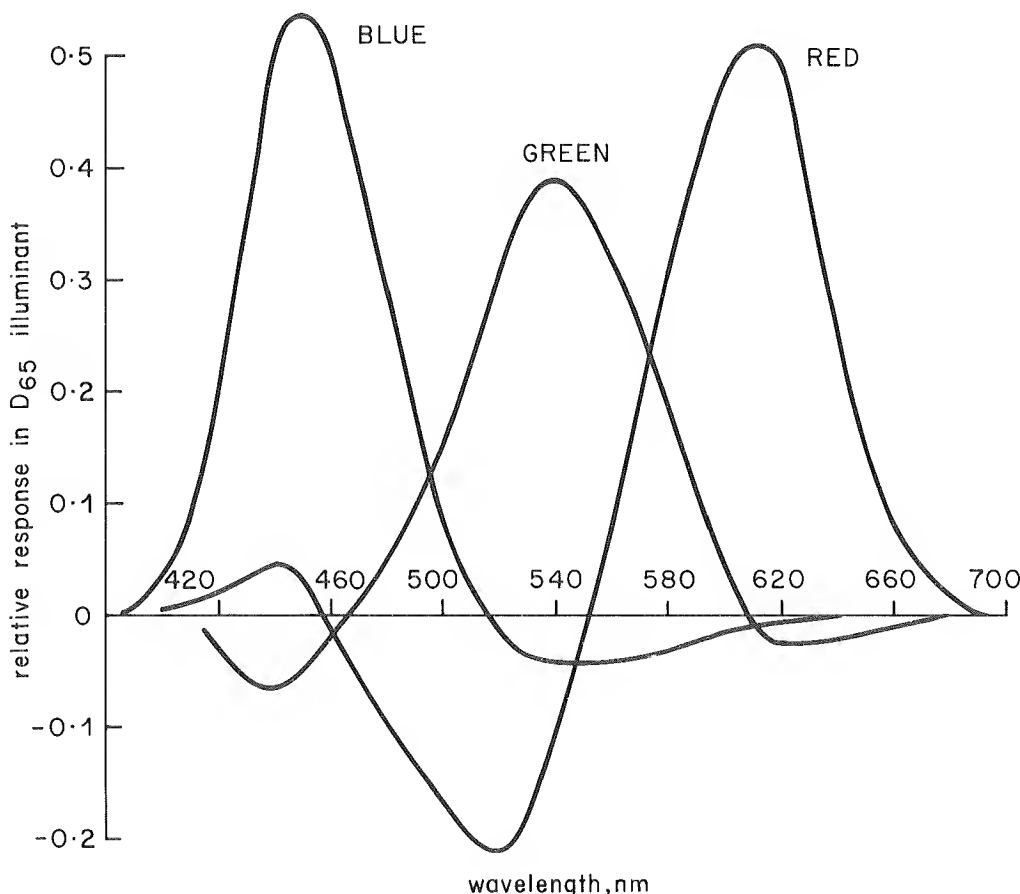

Fig. 5 - Ideal colour analysis of a camera designed for E.B.U. phosphors

ticities is shifted.

Consider as an example a shift in the green phosphor towards red; to compensate for this the receiver white-point setting would require a reduced red drive. Colours near to red would therefore suffer a loss of saturation, and colours near to green would be reproduced too red.

A correction matrix in linear signals would subtract a fraction of the green signal from the red channel and a fraction of the red signal from the green.

$$\begin{bmatrix} R_{OUT} \\ G_{OUT} \\ B_{OUT} \end{bmatrix} = \begin{bmatrix} 1+a & -a & 0 \\ -b & 1+b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} R_{IN} \\ G_{IN} \\ B_{IN} \end{bmatrix}$$

This would give perfect correction for all the colours common to the ideal and display-phosphor triangles.

For a general shift in all three phosphors, the required correction matrix can be derived by considering the equivalent matrix operations shown in Fig. 6. In (a) which represents the display with shifted primaries, the output values of U,V,W, should be identical to those obtained with an ideal display for the same values of red, green and blue drive signals. This equivalence is illustrated in (b) from which it is evident that correction matrix $[M]$ is:-

$$[M] = [D]^{-1} [I]$$

This matrix will give perfect correction within the area on the C.I.E. chromaticity diagram common to both ideal- and display-primary triangles as shown in Fig. 7. Colours included in I, but excluded from D, require negative amounts of the D primaries and therefore cannot be reproduced. Colours in D but not in I will not be reproduced correctly because "negative" drive signals would be required.
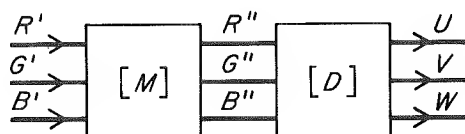
(a) display with shifted primaries and matrix correction to EBU display
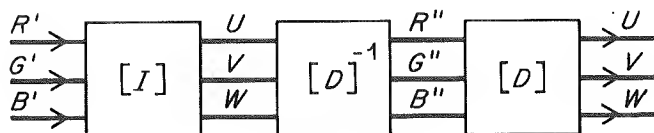
(b) equivalent arrangement to (a)

Fig. 6 - Linear matrix correction and equivalent matrix operations

$[M]$ = corrective matrix $[D]$ = display matrix

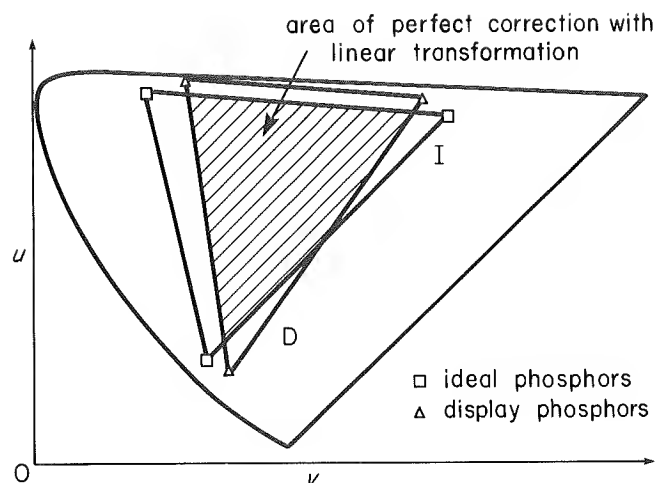$[I]$ = ideal display matrix

Fig. 7 - Phosphor chromaticity triangles on C.I.E. U.C.S. diagram

3.1. Direct transformation of gamma-corrected signals

It was believed that the use of a matrix with gamma-corrected R'.G'.B'. signals could offer a compromise solution since the corrections required are relatively small. The effect of matrix correction (calculated for linear signals) under such circumstances is to produce over-correction which increases with the colour saturation. The main-term co-efficients of the linear matrix in the case of saturated colours, are effectively increased to the exponent of the display-tube gamma, but the chromaticities cannot become more saturated than the phosphors permit. However the luminance components are also increased and, if these become excessive cause the more saturated colours to appear fluorescent. This suggests that the errors in saturated colours can be lessened by reducing the correction co-efficients of the matrix and poses the question of how the reductions should be made in order to provide the most acceptable overall result.

3.2. Weighted transformation

This line of thought gave rise to the weighted transformation,[8] which reduces the correction terms of the matrix by assuming that the display chromaticities are shifted only a fraction of the distance between the ideal and display primaries, as shown in Fig. 8. The matrix for the artificial primary set of phosphors is termed a percentage-shift matrix.

3.3. Optimised transformation

At best, a matrix operating on gamma-corrected signals will be a compromise and should be chosen to give minimum errors for a wide range of colours, with extra significance attached to skin-tones since these are generally considered to be more critical.

Such a matrix may be arrived at by means of a computer optimisation that minimises chrominance and luminance errors, using a range of test colours with
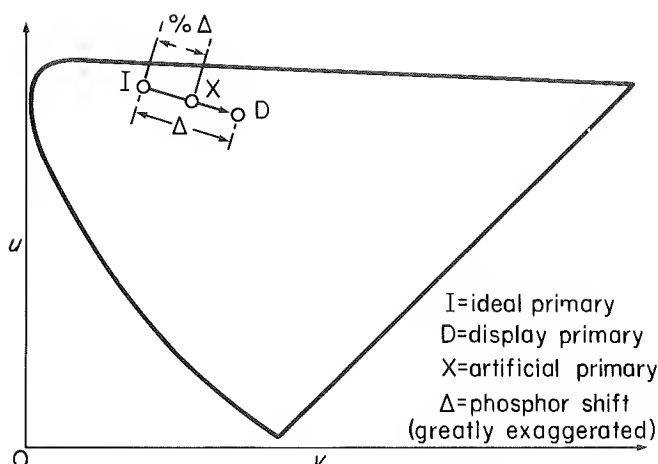
Fig. 8 - Derivation of artificial primaries

I=ideal primary
D=display primary
X=artificial primary
Δ=phosphor shift (greatly exaggerated)

additional weighting for skin-tones.

A computer programme was developed to compare a set of test colours as reproduced on a standard E.B.U. display with the same colours reproduced on a specified display, assuming the same $R'.G'.B'$. signals in each case. The programme has provision for applying a matrix to the linear or the gamma-corrected signals; it calculates chrominance and luminance errors and can optimise the matrix for minimum chrominance or minimum overall (luminance and chrominance) errors.

4. Matrix correction of different primaries

In Table I the chromaticities of four different phosphor sets are shown. These represent the ideal E.B.U. specifi-

cation, tubes specially made with primaries close to the E.B.U. specification, a currently used imported specimen tube, and the original N.T.S.C. specification. The primaries for these tubes are also plotted on a C.I.E. U.C.S. diagram in Fig. 9 which includes the E.B.U. tolerance boxes around the E.B.U. primaries.

4.1. The specimen tube

Correction matrices, shown in Table II, were calculated for the specimen tube, using direct and weighted transformations, and with the aim-point chromaticities set equal to those displayed on the E.B.U. tube.

The resultant chromaticity errors obtained with these matrices for twenty-six test colours and with E.B.U. tube display aim-points will be seen in Fig. 10, and in Fig. 11 which shows the skin-tone errors in greater detail. The locus of each of the displayed colours is shown as a function of matrix weighting; 0%* and 100% are indicated by the base and point of an arrow respectively. With no matrix correction (0%) the errors are produced solely by the display-primary chromaticities and appear as both desaturation effects and hue shifts, which are greater for the more saturated colours, particularly in the orange, red and magenta regions; skin-tones shift considerably towards green.

Contrary to the simplistic explanation of Section 3.1., the 100% matrix produces large hue errors rather than gross over-saturation; this is because the primaries are shifted in hue as well as saturation. The 100% matrix has been noted as giving gross over-saturation in the reds by de Marsh;[7] this work was however concerned with N.T.S.C. phosphors, involving much larger primary shifts.
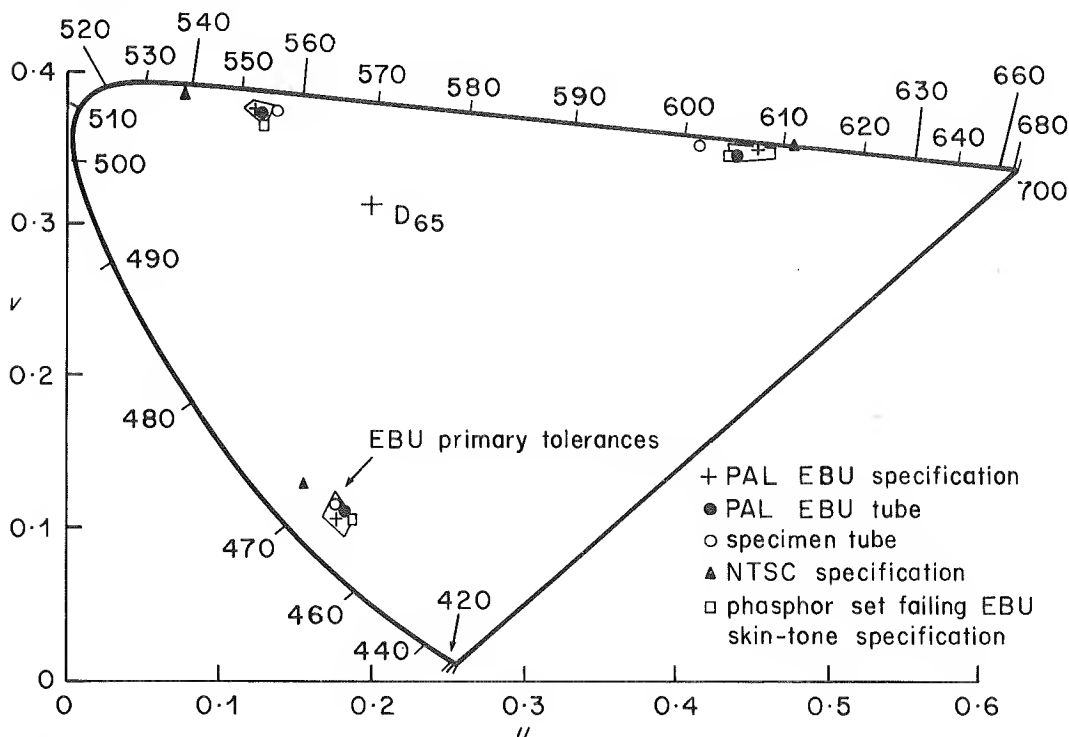
* i.e. no matrix correction

Fig. 9 - Chromaticity co-ordinates of shadow-mask tubes.

EBU primary tolerances

+ PAL EBU specification
● PAL EBU tube
o specimen tube
▲ NTSC specification
□ phosphor set failing EBU skin-tone specification

TABLE 1

Chromaticity co-ordinates of shadow-mask tubes

	RED		GREEN		BLUE		RED		GREEN		BLUE	
	x	y	x	y	x	y	u	v	u	v	u	v
P.A.L. E.B.U. specification	0·640	0·330	0·290	0·600	0·150	0·060	0·451	0·349	0·121	0·374	0·175	0·105
E.B.U. Tube	0·621	0·326	0·296	0·583	0·152	0·067	0·438	0·345	0·126	0·372	0·174	0·115
Specimen Tube	0·619	0·350	0·320	0·585	0·155	0·063	0·415	0·352	0·137	0·374	0·180	0·110
N.T.S.C. specification	0·670	0·330	0·210	0·710	0·140	0·080	0·477	0·352	0·076	0·384	0·152	0·130

TABLE II

Direct transformation matrices

Weighting

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1·11 & -0·10 & -0·01 \\ -0·03 & 1·02 & 0·01 \\ 0·01 & 0·03 & 0·96 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad 80\%$$

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1·15 & -0·13 & -0·02 \\ -0·03 & 1·02 & 0·01 \\ 0·02 & 0·03 & 0·95 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad 100\%$$

The chromaticity errors are reduced by corrective matrixing but at the expense of increased luminance errors. It will be seen from Fig. 12 that the corresponding luminance errors with no correction are small, but increase considerably for colours such as orange, red and pink when the correction is applied. However, this can be an advantage because even quite large luminance errors are usually subjectively less disturbing than numerically similar errors in chromaticity, except where excessively high luminance produces fluorescent effects.

The matrices shown in Table III optimised for minimum overall errors and for minimum chromaticity errors provide worthwhile improvements over the direct and weighted transformations, notably in the magentas and pinks, where the chromaticity errors are considerably reduced as shown in Fig. 10. The corresponding luminance errors, shown in Fig. 13, compared with those for direct and weighted transformation, shown in Fig. 12, are

TABLE III

Optimised transformation matrices

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1·04 & -0·06 & 0·02 \\ -0·10 & 1·05 & 0·05 \\ -0·02 & 0·03 & 0·99 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{Chromaticity error optimisation}$$

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1·08 & -0·04 & -0·04 \\ -0·03 & 1·01 & 0·02 \\ 0·01 & 0·04 & 0·95 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{Overall error optimisation}$$

0·38 GREEN

4

5

0·34 3 11 12 13 RED

skin
tones

0·30 + D₆₅ 7 15

10

2

0·26 9 16

• chromaticity aim point

8

0·22 0% 100% direct transformation
80%

+ optimised transformation (chromaticity)

0·18 o optimised transformation (overall)

□ EBU tube primaries

1

0·14 △ specimen tube primaries

BLUE

0·10
0·08 0·12 0·16 0·20 0·24 0·28 0·32 0·36 0·40 0·44 0·48
u
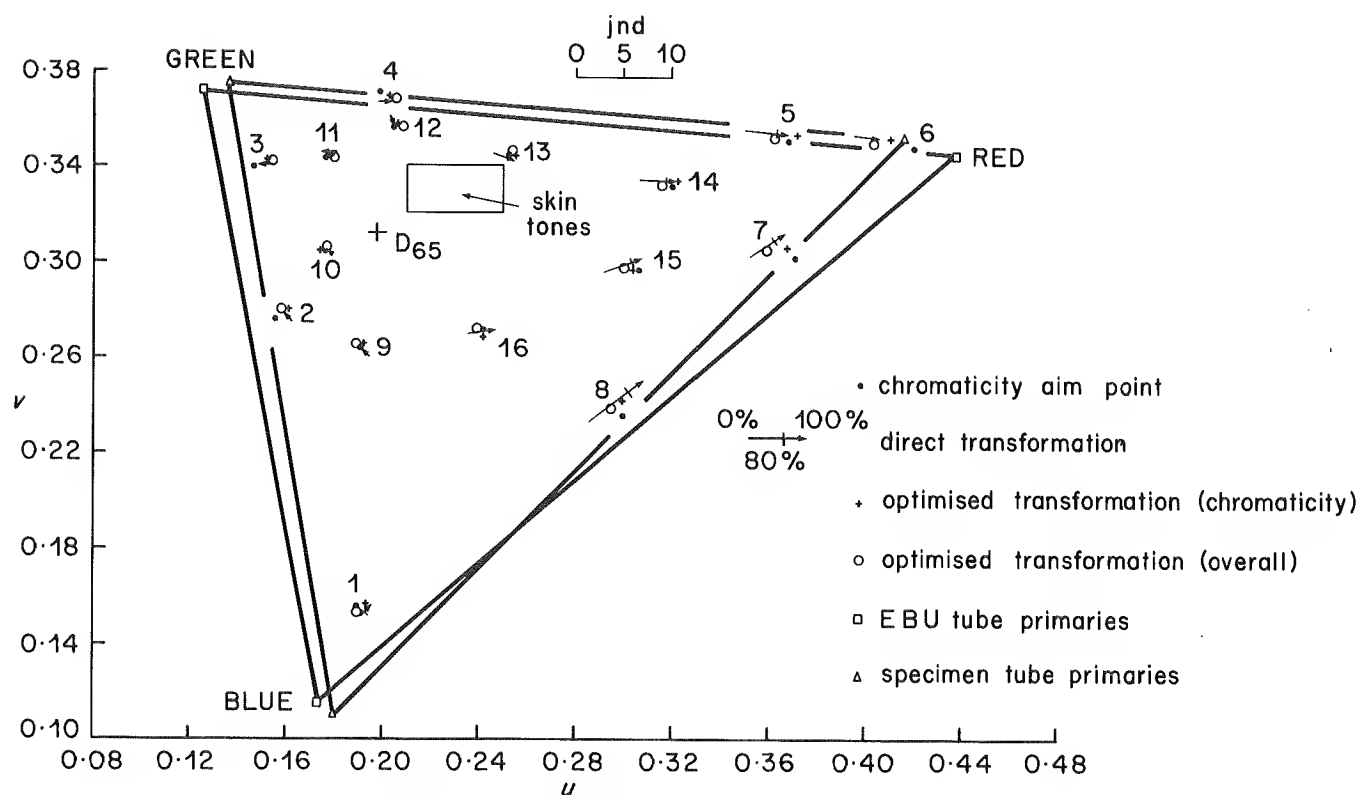
Fig. 10 - Chromaticity errors, specimen display tube

0·34

• chromaticity aim point

0%   100% direct transformation
80%

+ optimised transformation (chromaticity)

o optimised transformation (overall)

20

19

21

0·33 23

18 24

17,22

0 jnd 1

0 1

0 1 approximate adjustment for skin-tone

0·32
0·22 0·23 0·24
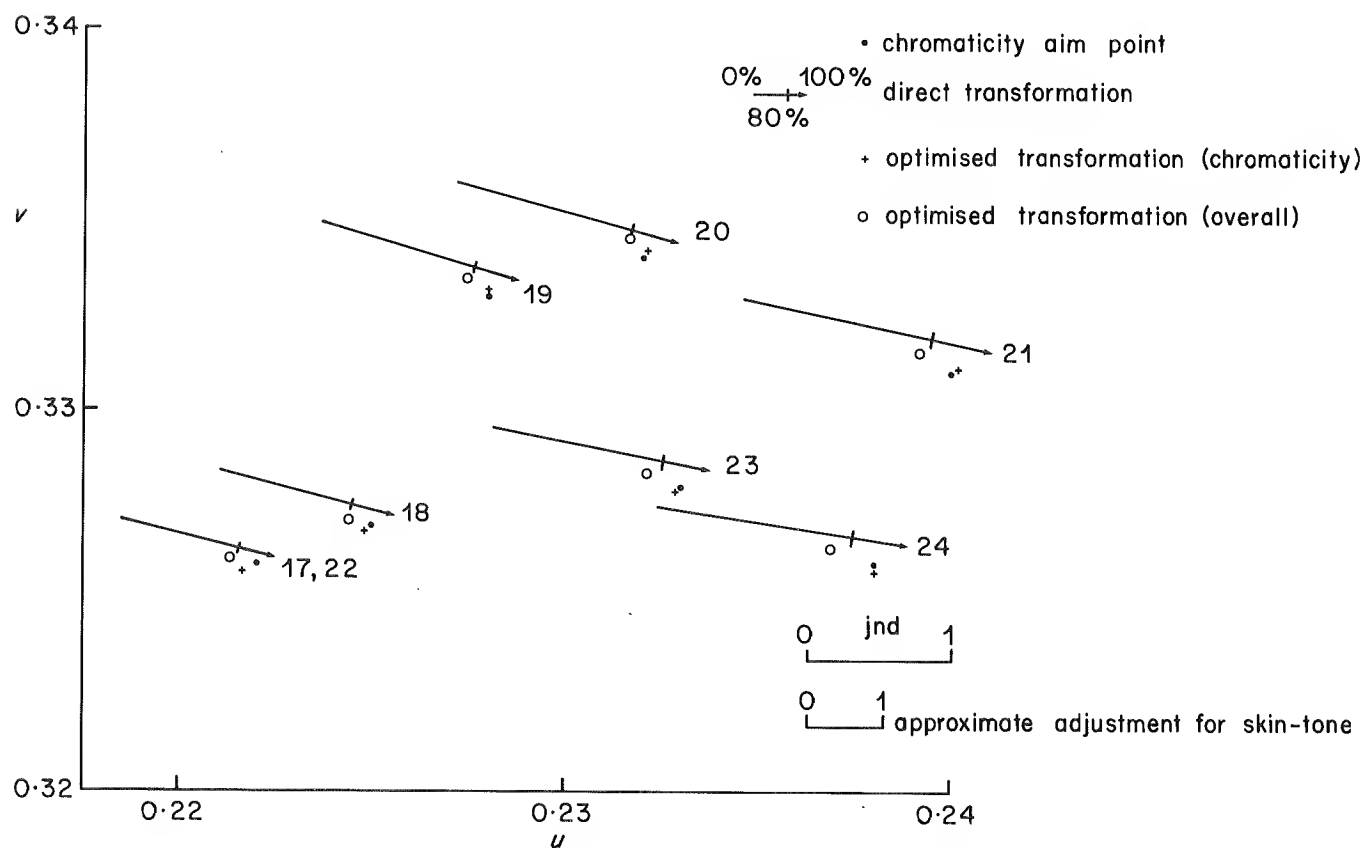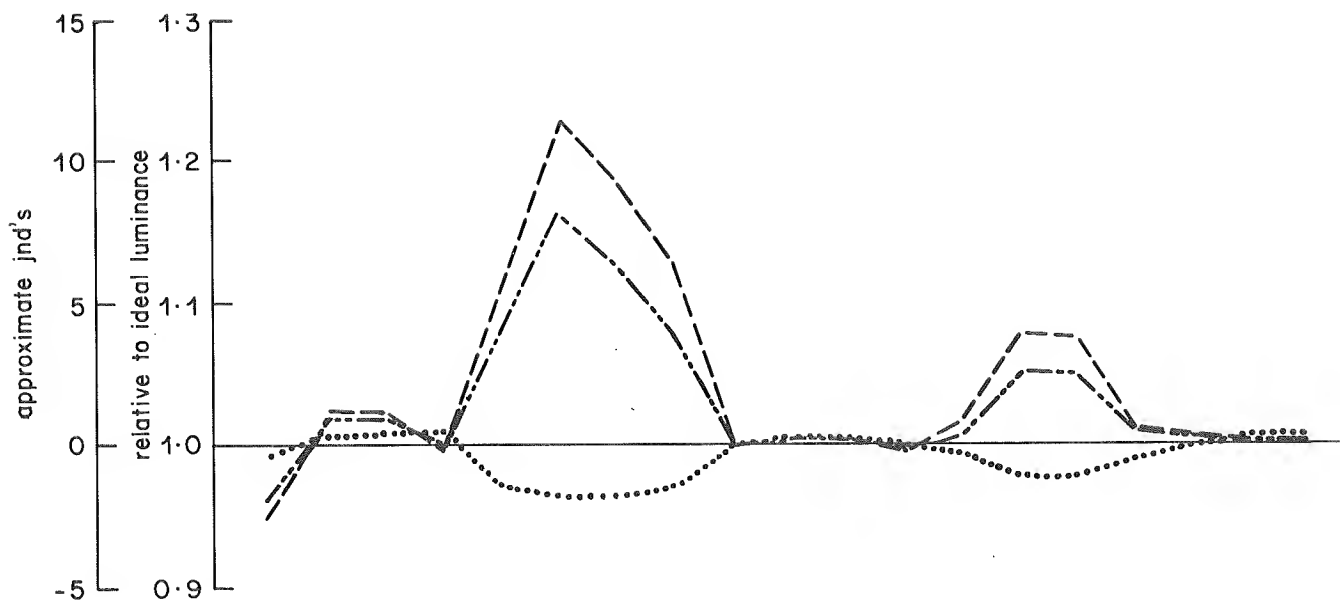u

Fig. 11 - Chromaticity errors for skin-tones — specimen display tube

colour	1 blue	2 cyan	3 green	4 yellow	5 orange	6 red	7 pink	8 magenta	9 blue	10 cyan	11 green	12 yellow	13 orange	14 red	15 pink	16 magenta	17-24 skin	25 grass	26 grass
ideal luminance (relative to white)	7·0	45	50	65	25	15	20	12	32	62	48	60	40	12	14	30	28	25	7
	high saturation								lower saturation										

Fig. 12 - Luminance errors for all the test colours — specimen tube, direct and weighted transformation

— — ·· — 80% direct transformation — — — — 100% direct transformation •••••••• no correction

TABLE IV

Chromaticity and luminance errors for specimen tube in j.n.d.'s

MATRIX	CHROMATICITY ERROR	LUMINANCE ERROR	TOTAL R.M.S. ERROR	SKIN TONE CHROM	SKIN TONE LUMINANCE	SKIN TONE R.M.S. TOTAL
Unity matrix	1·77	0·48	1·83	1·25	0·27	1·28
80% weighted transformation	0·66	1·07	1·26	0·20	0·17	0·26
100% direct transformation	0·66	1·50	1·64	0·21	0·33	0·39
Optimisation for overall error	0·71	0·78	1·05	0·19	0·14	0·23
Optimisation for chrom error	0·53	2·56	2·61	0·08	2·66	2·66

colour	1 blue	2 cyan	3 green	4 yellow	5 orange	6 red	7 pink	8 magenta	9 blue	10 cyan	11 green	12 yellow	13 orange	14 red	15 pink	16 magenta	17-24 skin	25 grass	26 grass
ideal luminance (relative to white)	7.0	45	50	65	25	15	20	12	32	62	48	60	40	12	14	30	28	25	7
	high saturation								lower saturation										

Fig. 13 - Luminance errors for all the test colours — specimen tube, optimised transformations

———————— optimised transformation (chromaticity)　　　— — — — optimised transformation (overall)　　　•••••• no correction

similar but reduced with overall optimisation. With chrominance optimisation the errors are, on average, greater but more widely distributed amongst the test colours.

In the expanded plot of skin-tones, Fig. 11, it will be seen that the minimum errors are obtained with chromaticity optimisation. This is also shown, with the results for all the test colours, in Table IV which is a summary of the errors obtained with the different correction matrices shown in Tables II and III.

4.2. N.T.S.C. phosphors

Similarly, correction matrices shown in Table V were calculated for a display with N.T.S.C. phosphors, as specified in Table I, and with chromaticity aim-points corresponding to an E.B.U. tube.

The chromaticity errors resulting with these correction matrices are shown in Fig. 14 with, as previously, the locus of the displayed colours for direct and weighted transformations from 0% to 100%, (at the base and point of the arrow respectively). It will be seen that the weighted transformation gives over-saturation at 0%, with chrominance errors reducing to a minimum around 85%, in keeping with results obtained by C.B. Neal[7] for the

converse problem. Again, the optimisations give worthwhile improvements, particularly in magenta, and result in the smallest r.m.s. errors for skin-tones which will be seen in the expanded plot, Fig. 15. Corresponding luminance errors for all the test colours are shown in Fig. 16 and a summary of all the performances of the correction matrices for the N.T.S.C. phosphors are detailed in Table VI.

Correction in this case gives a poorer final result, because the phosphor shifts are so much larger than in the previous example, giving larger correction co-efficients, which are effectively increased by gamma. The shifts are also in the opposite direction as the N.T.S.C. display has a much larger gamut than the reference display; this reduces the restriction imposed by the intrinsic display primaries since almost all the colours in the gamut of the reference display can be reproduced by the N.T.S.C. phosphors.

4.3. Phosphor set failing to meet the E.B.U. skin-tone test

As mentioned in Section 1, a set of tolerances on the display primary chromaticities has been agreed by the E.B.U.[3] and are based on the intrinsic colour of the primaries being such as to give good colorimetry while still allowing for reasonable manufacturing spreads.

TABLE V

Correction matrices for N.T.S.C. phosphors

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 0 \cdot 76 & 0 \cdot 22 & 0 \cdot 02 \\ 0 \cdot 02 & 1 \cdot 03 & -0 \cdot 05 \\ 0 \cdot 01 & 0 \cdot 05 & 0 \cdot 94 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{80\% direct}$$

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 0 \cdot 72 & 0 \cdot 25 & 0 \cdot 03 \\ 0 \cdot 02 & 1 \cdot 04 & -0 \cdot 06 \\ 0 \cdot 02 & 0 \cdot 06 & 0 \cdot 92 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{100\% direct}$$

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 0 \cdot 77 & 0 \cdot 17 & 0 \cdot 06 \\ 0 \cdot 03 & 1 \cdot 02 & -0 \cdot 05 \\ 0 \cdot 06 & 0 \cdot 07 & 0 \cdot 87 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{Overall optimisation}$$

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 0 \cdot 84 & 0 \cdot 15 & 0 \cdot 01 \\ 0 \cdot 12 & 0 \cdot 93 & -0 \cdot 05 \\ 0 \cdot 12 & 0 \cdot 05 & 0 \cdot 83 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \quad \text{Chromaticity optimisation}$$

TABLE VI

Chromaticity and luminance errors for N.T.S.C. phosphors, in j.n.d.'s

MATRIX	CHROMATICITY ERROR	LUMINANCE ERROR	TOTAL R.M.S. ERROR	SKIN TONE CHROM ERROR	SKIN TONE LUM ERROR	SKIN TONE R.M.S. TOTAL
Unity	4·84	3·88	6·21	4·29	1·78	4·64
80% weighted transformation	1·42	1·47	2·04	0·75	0·06	0·76
100% direct transformation	1·18	1·98	2·31	0·34	0·20	0·39
Optimisation for overall error	1·13	1·35	1·76	0·15	0·34	0·37
Optimisation for chrom error	1·04	3·28	3·45	0·13	3·02	3·02

The tolerances consist of boxes surrounding each of the System I primaries as shown on the chromaticity diagram in Fig. 9. A phosphor set is acceptable if its chromaticities fall inside these boxes and a specified skin-tone is reproduced with less than 0·003 units of error on the u,v diagram. ($\approx 0 \cdot 8$ j.n.d.). Combinations of display primaries which fall inside the boxes but do not meet the skin-tone test can be made acceptable by matrixing.

One such set of phosphors with chromaticity co-ordinates as detailed in Table VII and failing to meet the E.B.U. requirements in this respect, was chosen and result in a chrominance error for the skin-tone test of 0·0035 uv units (or ≈ 1 j.n.d.).

Correction matrices, shown in Table VIII, were calculated for this set based on chromaticity aim-points displayed on a tube with true E.B.U. primaries.
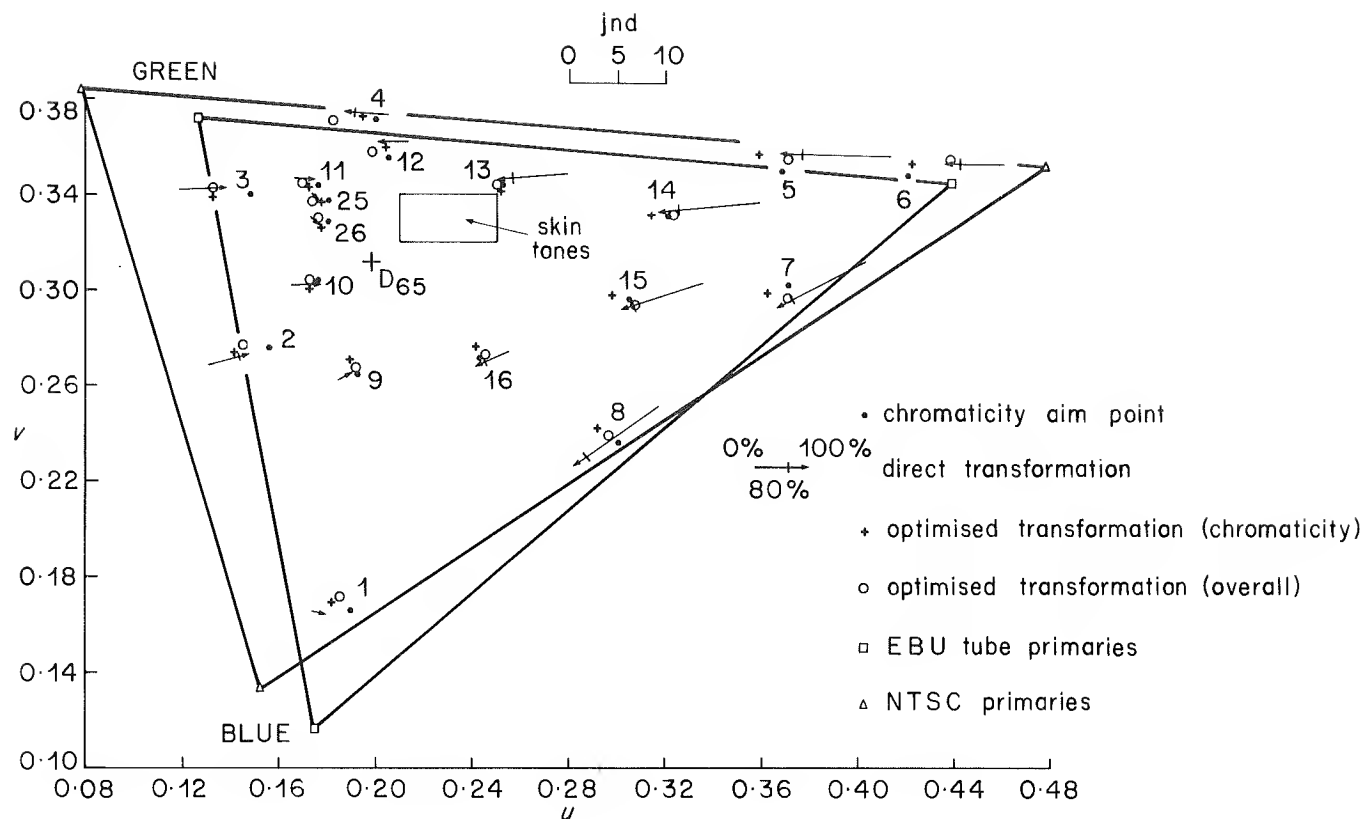
(PH-172)
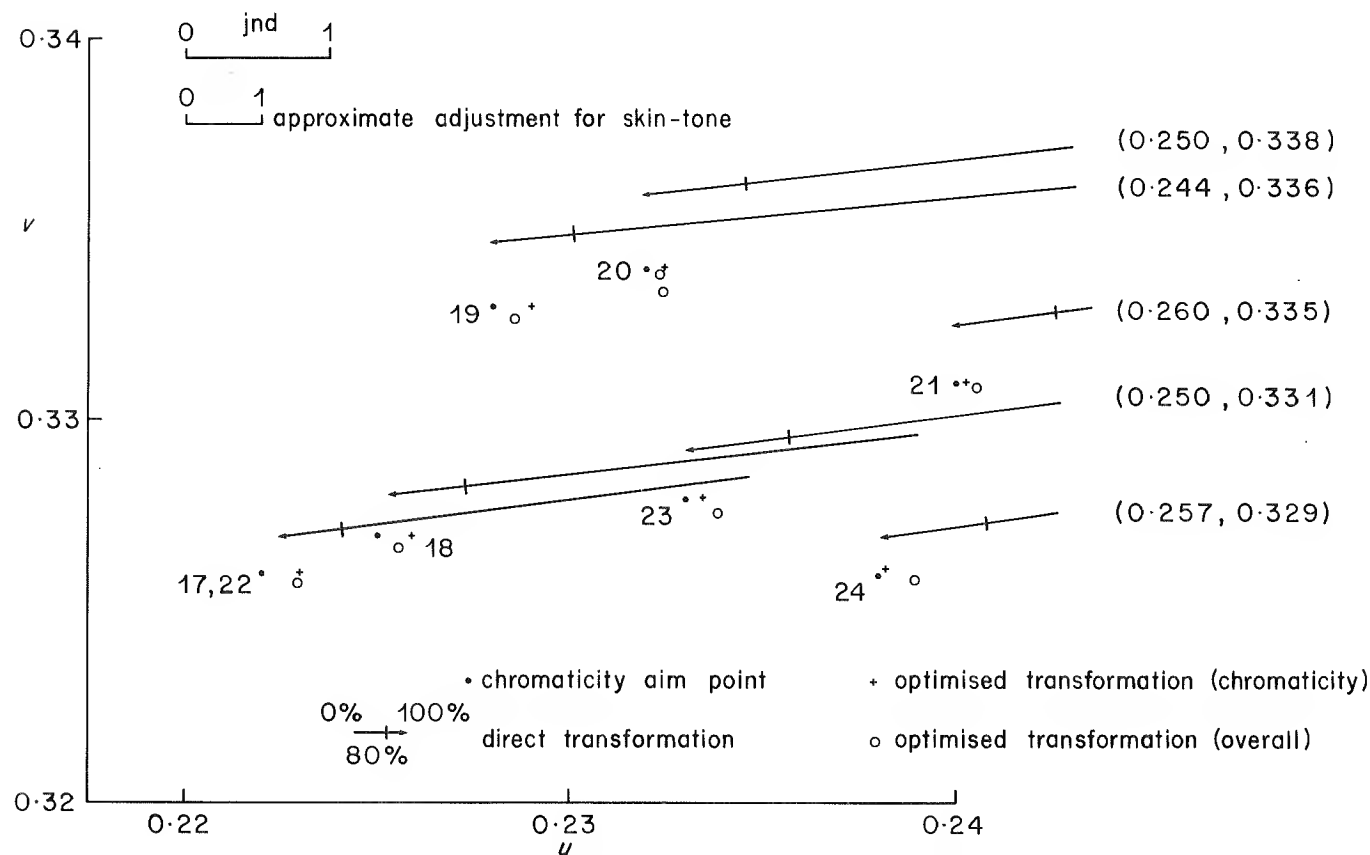
— 11 —

Fig. 14 - Chromaticity errors — N.T.S.C. primaries

Fig. 15 - Chromaticity errors for skin-tones — N.T.S.C. primaries

Chromaticity co-ordinates of phosphor-set failing E.B.U. face-tone test

	u	v	x	y
Red	0·432	0·343	0·611	0·324
Green	0·128	0·363	0·284	0·537
Blue	0·185	0·106	0·158	0·060

The error for the E.B.U. specified skin-tone test, shown in Fig. 17 is reduced to <0·04 j.n.d. by the optimised matrices. It will be seen from Table IX that in this case, there is only a small overall improvement compared with the direct transformation, as would be expected from the small correction co-efficients.

5. Discussion

The weighted transformation method, amounts to an optimisation by hand of one variable, the co-efficient of the transformation. A computer optimisation on the other hand operates on six independent variables and offers the possibility of weighting errors in each test colour according to their perceptibility. From all the previous examples
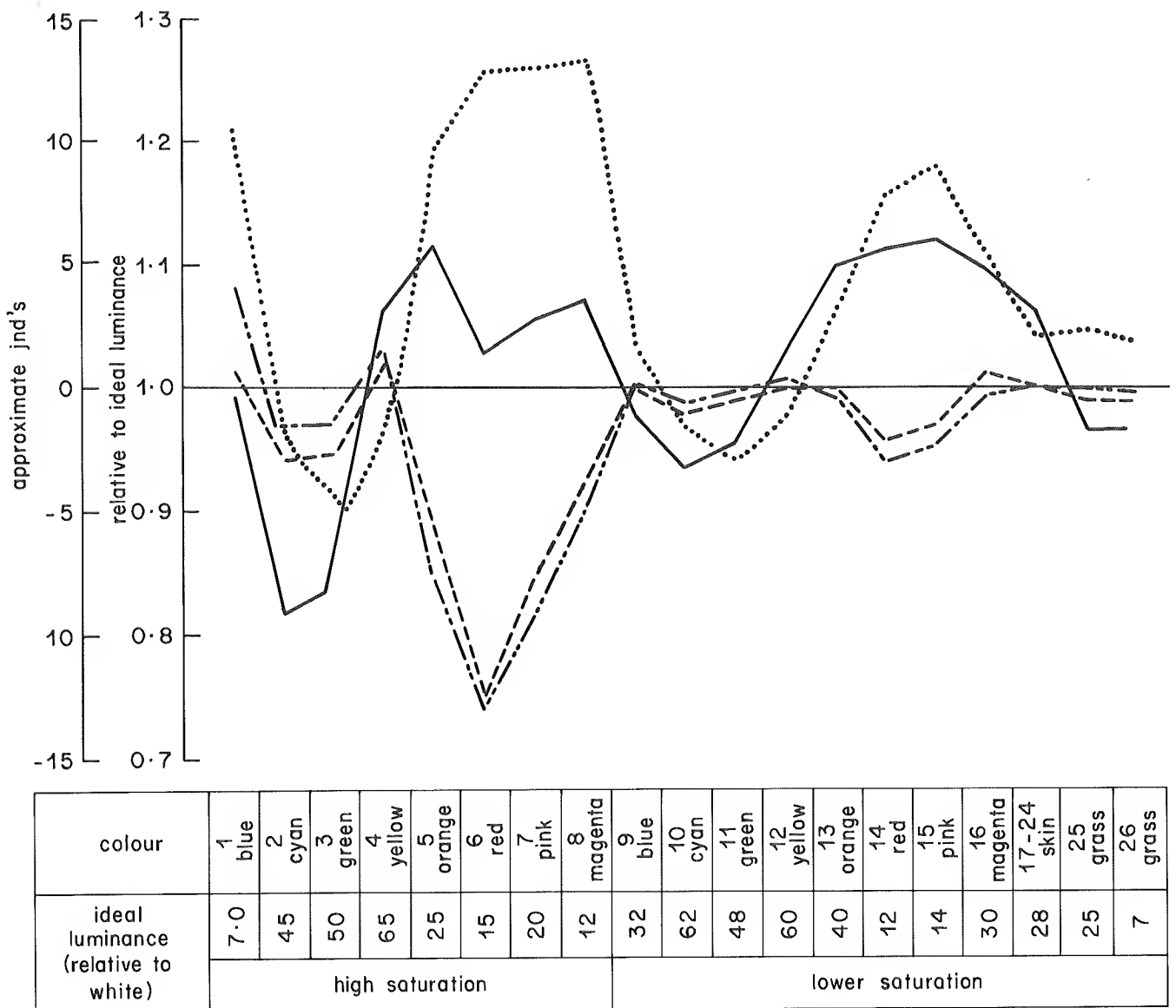

colour	1 blue	2 cyan	3 green	4 yellow	5 orange	6 red	7 pink	8 magenta	9 blue	10 cyan	11 green	12 yellow	13 orange	14 red	15 pink	16 magenta	17-24 skin	25 grass	26 grass
ideal luminance (relative to white)	7·0	45	50	65	25	15	20	12	32	62	48	60	40	12	14	30	28	25	7
	high saturation								lower saturation										

Fig. 16 - Luminance errors for all the test colours — N.T.S.C. primaries

——————— optimised transformation (chromaticity)   — — — — optimised transformation (overall)

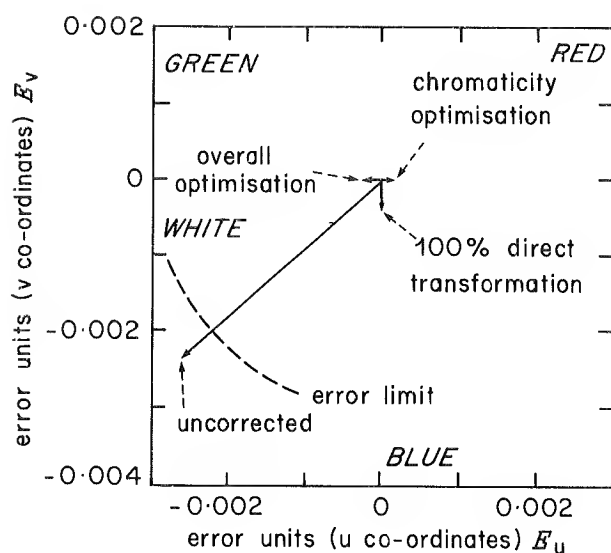•••••••• no correction   —•—•—•— 80% direct transformation

Fig. 17 - Chromaticity errors — departure from E.B.U. specified skin-tone (u = 0·222, v = 0·326) point

it appears that this technique also results in the smallest errors, particularly when the phosphor sets with larger primary-colour shifts are involved and require larger correction-matrix co-efficients. Practical tests carried out with similar matrices indicate that, subjectively, the use of chromaticity-error optimisation produces the most acceptable results.

It is accepted practice to use tubes with identical phosphors under conditions of side-by-side comparison. It is therefore not envisaged that matrix correction would be used to allow tubes with non-E.B.U. primaries to appear alongside E.B.U. displays, as the residual chromaticity errors would be visible to a critical viewer.

The example of Section 4.1. shows acceptable errors over most of the chromaticity diagram, see Fig. 12, the only serious shortcoming is the reduction in the gamut of reproducible colours. In this case the result of such a reduction is that some frequently encountered saturated

reds cannot be accurately reproduced. It is difficult to specify limits on a phosphor set that has a gamut greater than that of the standard. If corrected non-E.B.U. displays were used in isolation, a criterion of colour fidelity for saturated colours similar to that specified for colour-camera appraisal might be satisfactory and would mean that chrominance and luminance errors for saturated colours should be less than 5 j.n.d; skin-tone performance should conform to the E.B.U. test.

6. Conclusions

Compared with weighted direct transformation, the computer optimisation technique offers the ability to weight correction for particular colours such as skin-tones and provides improved overall correction, particularly when the phosphor-primary shifts are considerable. Subjectively, optimisation based on minimising chromaticity errors produces the best results.

It is almost certain that all combinations of phosphors with chromaticities lying within the E.B.U. tolerance boxes can be corrected where necessary by this means to give a satisfactory performance in the skin-tone test.

While it is recommended that the limits of the E.B.U. specified colour-gamut are adhered to, it has been shown that some phosphor sets lying outside the boxes can be corrected to give good colorimetry over most of the gamut common to both sets. A reduction in the gamut within that specified by the E.B.U. should, if possible, be avoided; on the other hand the nature of the problem precludes any rigid limits to be fixed on the maximum permissible increase in gamut. Thus the suitability of a phosphor set for correction should be assessed in each case by calculation and observation.

7. References

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TABLE VIII

Correction matrices for phosphor set failing E.B.U. skin-tone test

$$
\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1\cdot08 & -0\cdot04 & -0\cdot04 \\ -0\cdot01 & 1\cdot00 & 0\cdot01 \\ -0\cdot03 & -0\cdot13 & 1\cdot16 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}
$$ 100% direct transformation

$$
\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1\cdot05 & -0\cdot02 & -0\cdot03 \\ -0\cdot02 & 1\cdot00 & 0\cdot02 \\ -0\cdot06 & -0\cdot07 & 1\cdot13 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}
$$ Overall optimisation

$$
\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1\cdot08 & -0\cdot06 & -0\cdot02 \\ -0\cdot01 & 1\cdot00 & 0\cdot01 \\ -0\cdot06 & -0\cdot12 & 1\cdot18 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix}
$$ Chromaticity optimisation

TABLE IX

Chromaticity and luminance errors for phosphor set failing E.B.U. face tone test in j.n.d.'s

MATRIX	CHROMATICITY ERROR	LUMINANCE ERROR	TOTAL R.M.S. ERROR	SKIN TONE CHROM ERROR	SKIN TONE LUM ERROR	SKIN TONE R.M.S. TOTAL
Unity matrix	1·57	0·48	1·64	1·12	0·06	1·12
100% direct transformation	0·52	0·82	0·97	0·07	0·20	0·21
Optimisation for overall error	0·60	0·42	0·73	0·12	0·07	0·14
Optimisation for chrom error	0·52	0·80	0·95	0·06	0·09	0·11

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8. APPENDIX

8.1. Correction by matrixing colour-difference signals

The matrix relationships considered in Section 4 are of the form

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} r_R & r_G & r_B \\ g_R & g_G & g_B \\ b_R & b_G & b_B \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \qquad (8.1)$$

where

$$\left. \begin{array}{rcl} r_R + r_G + r_B &=& 1 \\ g_R + g_G + g_B &=& 1 \\ b_R + b_G + b_B &=& 1 \end{array} \right\} \qquad (8.2)$$

When a coded transmission system is used the primary signals R', G' and B' are derived, in the decoder, from a wide-band luminance signal Y' and three narrow-band colour-difference signals

$$\left. \begin{array}{rcl} \Delta'_R &=& (R' - Y') \\ \Delta'_G &=& (G' - Y') \\ \text{and} \quad \Delta'_B &=& (B' - Y') \end{array} \right\} \qquad (8.3)$$

i.e. $R' = \Delta'_R + Y'$, etc.

from which

$$\begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} = \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} + \begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix} \qquad (8.4)$$

Let Δ''_R, Δ''_G and Δ''_B be defined as modified colour-difference signals which, when added in turn to the original luminance signal Y' produce the same R'', G'' and B'' as does the matrix operation expressed in Equation 8.1.

i.e.
$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} + \begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix} \qquad (8.5)$$

Substituting the relationships (8.4) and (8.5) into (8.1) gives

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} + \begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix} = \begin{bmatrix} r_R & r_G & r_B \\ g_R & g_G & g_B \\ b_R & b_G & b_B \end{bmatrix} \left\{ \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} + \begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix} \right\}$$

$$= \begin{bmatrix} r_R & r_G & r_B \\ g_R & g_G & g_B \\ b_R & b_G & b_B \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} + \begin{bmatrix} r_R & r_G & r_B \\ g_R & g_G & g_B \\ b_R & b_G & b_B \end{bmatrix} \begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix}$$

From Equation (8.2), the second component of the right-hand side of this is equal to

$$\begin{bmatrix} Y' \\ Y' \\ Y' \end{bmatrix}$$

which may be subtracted from both sides of the equation giving

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} = \begin{bmatrix} r_R & r_G & r_B \\ g_R & g_G & g_B \\ b_R & b_G & b_B \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} \qquad (8.6)$$

This is of the same form as Equation (8.1), i.e. a chrominance correction that can be achieved by matrixing the primary signals $R'G'B'$ and could equally have been achieved by applying the same matrix relationship to the colour difference signals. There is, however, one very important difference, in that whereas Equation (8.1) is a unique relationship Equation (8.6) is only one of a family of relationships with the same final result. This can be shown by noting that the definition of Y'

$$Y' = 0.299R' + 0.587G' + 0.114B'$$

leads to the relationship $0.299 \Delta'_R + 0.587 \Delta'_G + 0.114 \Delta'_B = 0$ so that, while R', G' and B' are independent quantities, Δ'_R Δ'_G and Δ'_B are not independent. From this, if l, m, n are any three independent constants, then

$$\begin{bmatrix} 0.299l & 0.587l & 0.114l \\ 0.299m & 0.587m & 0.114m \\ 0.299n & 0.587n & 0.114n \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} = 0 \qquad (8.7)$$

Adding this zero to the right-hand side of Equation 8.6.

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} = \begin{bmatrix} (r_R + 0.299l) & (r_G + 0.587l) & (r_B + 0.114l) \\ (g_R + 0.299m) & (g_G + 0.587m) & (g_B + 0.114m) \\ (b_R + 0.299n) & (b_G + 0.587n) & (b_B + 0.114n) \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} \qquad (8.8)$$

Which may be written

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} = \begin{bmatrix} r'_R & r'_G & r'_B \\ g'_R & g'_G & g'_B \\ b'_R & b'_B & b'_B \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} \qquad (8.9)$$

where $r'_G = (r_G + 0.299l)$, etc.

Thus any of the r' co-efficients can be made to take any required value by a suitable choice of l, any g' can be fixed by a suitable choice of m and any b' by a suitable choice of n. This gives considerable flexibility in the choice of matrix parameters, as will be shown in the following examples.

A matrix quoted in Section 4, Table III is

$$\begin{bmatrix} R'' \\ G'' \\ B'' \end{bmatrix} = \begin{bmatrix} 1.04 & -0.06 & 0.02 \\ -0.10 & 1.05 & 0.05 \\ -0.02 & 0.03 & 0.99 \end{bmatrix} \begin{bmatrix} R' \\ G' \\ B' \end{bmatrix} \qquad (8.10)$$

It would be possible to derive the modified colour difference signals using the same relationship but this would involve all three colour-difference signals and three of the matrix elements are negative, requiring polarity-reversing stages in the signal processing.

If the parameters are chosen so that r'_G, g'_G and b'_G are zero the relationship does not involve the original Green colour difference signal, which is not normally decoded directly but is derived from the other two. Putting $r'_G = 0$ requires

$$-0.06 + 0.587l = 0$$
$$l = 0.102$$

with similar results for m and n, from which

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} = \begin{bmatrix} 1.07 & 0.03 \\ -0.63 & -0.15 \\ -0.04 & 0.98 \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_B \end{bmatrix} \qquad (8.11)$$

Another possibility is to choose the parameters l, m, n so that one co-efficient in each row of the matrix is zero and the others positive. Once again $l = 0.102$, but $m = 0.10/0.299 = 0.33$ and $n = 0.02/0.299 = 0.07$. The matrix equation is then

$$\begin{bmatrix} \Delta''_R \\ \Delta''_G \\ \Delta''_B \end{bmatrix} = \begin{bmatrix} 1.07 & 0 & 0.03 \\ 0 & 1.25 & 0.09 \\ 0 & 0.07 & 1.00 \end{bmatrix} \begin{bmatrix} \Delta'_R \\ \Delta'_G \\ \Delta'_B \end{bmatrix} \qquad (8.12)$$

A further possibility is to derive Δ''_R Δ''_G and Δ''_B directly from the demodulated chrominance signals, i.e. E'_U and E'_V for a P.A.L. system, E'_R and E'_B or E'_I and E'_Q for N.T.S.C., or D'_R and D'_B for S.E.C.A.M.

The advantages of applying the matrix to the colour difference signals rather than to the $R'G'B'$ signals are that the operation is carried out on narrow-band signals, and that there is considerable flexibility in the choice of matrix.

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